

# Fibonacci, phi and magic reciprocals

L. W. Ellerbruch  
Northern Michigan University  
<http://Ellerbruch.nmu.edu>

## Resources

- 1 This Power Point presentation
- 1 pdf files for the content of this presentation
- 1 Spreadsheets in excel for the material in this presentation

FOR MORE INFO...

<http://Ellerbruch.nmu.edu/fibetc/maazonal.html>

## Exercises

- 1 Develop a spreadsheet showing the Fibonacci sequence and phi
- 1 Derive phi algebraically
- 1 Extend phi properties to other magic reciprocals
- 1 Develop a spreadsheet for magic reciprocals
- 1 Modify and extend the Fibonacci sequence

## Numerical Generation of Fibonacci sequence and phi

This is done easily using a spreadsheet:

<http://Ellerbruch.nmu.edu/fibetc/Fibonacci1.xls>

## Algebraic derivation of phi

$$x = \phi$$

$$1/x = x - 1$$

$$x^2 - x - 1 = 0$$

$$x = (1 \pm \sqrt{5})/2$$

thus  $x = 1.61803398875\dots$  or  $x = -.61803398875\dots$

and we can associate  $\phi = 1.61803398875\dots$

$$1/\phi = .61803398875\dots$$

## Algebraic Extension 1

$$1/x = x - 2$$

$$x^2 - 2x - 1 = 0$$

$$x = (2 \pm \sqrt{8})/2$$

thus  $x = 2.41421356237\dots$

and  $1/x = .41421356237\dots$

## Algebraic Extension 2

$$1/x = x - 3$$

$$x^2 - 3x - 1 = 0$$

$$x = (3 + \sqrt{13})/2$$

thus  $x = 3.30277563773\dots$

and  $1/x = .30277563773\dots$

## Algebraic Generalization

$$\forall \quad 1/x = x - b$$

- $x^2 - b x - 1 = 0$

- $x = (b + \sqrt{b^2 + 4})/2$

- Now a table in a spreadsheet will be useful.

To see the spreadsheet..

<http://Ellerbruch.nmu.edu/fibetc/magic.xls>

## Fibonacci Extension 1

Start the Fibonacci sequence with numbers other than 1

Download this excel spreadsheet and change the beginning values.  
Watch the ratio.

<http://Ellerbruch.nmu.edu/fibetc/Fibonacci1.xls>

## Fibonacci Extension 2

Sum more than two numbers in a Fibonacci like sequence.

Download this spreadsheet and vary the first three values.

<http://Ellerbruch.nmu.edu/fibetc/Fibonacci2.xls>

## Fibonacci Extension 3

Sum four numbers in a Fibonacci like sequence.

Download this spreadsheet and vary the first four values.

<http://Ellerbruch.nmu.edu/fibetc/Fibonacci3.xls>